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FUNCTIONAL EQUATIONS OF ITERATED INTEGRALS WITH REGULAR SINGULARITIES

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§0. Introduction

Polylogarithmic functions satisfy functional equations. The most famous equation is of course the functional equation of the logarithm

$$\log x + \log y = \log(x \cdot y).$$

The other well known equation is the Abel equation of the dilogarithm

$$\mathrm{Li}_2\Big(\frac{x}{1-x}\cdot\frac{y}{1-y}\Big) = \mathrm{Li}_2\Big(\frac{y}{1-x}\Big) + \mathrm{Li}_2\Big(\frac{x}{1-y}\Big) - \mathrm{Li}_2(x) - \mathrm{Li}_2(y) - \log(1-x)\log(1-y).$$

Polylogarithms are special cases of more general iterated integrals. One can hope that the known results about functional equations of polylogarithms hold also for more general iterated integrals. In fact in [5] we have proved some general results about functional equations of iterated integrals on $P^1(\mathbf{C})$ minus several points. In this paper we generalize our results from [5] to functional equations of iterated integrals on any smooth, quasi-projective algebraic variety.

Our principal tool is the universal unipotent connection with logarithmic singularities. First we prove our results for a complement of a divisor with normal crossings in a smooth, projective variety. Next, using results of Hironaka about resolution of singularities we extend our results to smooth, quasi-projective varieties. The proofs (for a complement of a divisor with normal crossings) are straightforward generalizations of methods from [5]. These results are in the first three sections of this paper.

In the fourth section of the paper we are dealing with the dilogarithm. It is well known that any functional equation of the logarithm on $P^1(\mathbf{C})$ can be obtained by successive applications of the functional equation

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