N. Obata and N.J. Wildberger
Nagoya Math. J.
Vol. 142 (1996), 67–93

GENERALIZED HYPERGROUPS AND ORTHOGONAL POLYNOMIALS

NOBUAKI OBATA AND NORMAN J. WILDBERGER

Introduction

We study in this paper a generalization of the notion of a discrete hypergroup with particular emphasis on the relation with systems of orthogonal polynomials. The concept of a locally compact hypergroup was introduced by Dunkl [8], Jewett [12] and Spector [25]. It generalizes convolution algebras of measures associated to groups as well as linearization formulae of classical families of orthogonal polynomials, and many results of harmonic analysis on locally compact abelian groups can be carried over to the case of commutative hypergroups; see Heyer [11], Litvinov [17], Ross [22], and references cited therein. Orthogonal polynomials have been studied in terms of hypergroups by Lasser [15] and Voit [31], see also the works of Connett and Schwartz [6] and Schwartz [23] where a similar spirit is observed.

The special case of a discrete hypergroup, particularly in the commutative case, goes back earlier. In fact the ground-breaking paper of Frobenius [9] implicitly uses the notion of a hypergroup as the central object upon which the entire edifice of harmonic analysis on a finite (non-abelian) group is built (see Curtis [7] for an interesting discussion of this important historical point, and also Wildberger [34] for an extension of this point of view to Lie groups). Variants of the concept have appeared in many places: the early work of Kawada [13] on C-algebras; the systems of generalized translation operators studied by Levitan [16]; the hypercomplex systems studied by Berezansky and Kalyushnyi [4] and others; the work of the physicists on Racah-Wigner algebras (see for example Sharp [24]); the association schemes studied by combinatoricists (see for example the book of Bannai and Ito [3]); and the work of McMullen [18] and McMullen and Price [19]. More recently we mention also the objects introduced by Arad and Blau [1] called table algebras (see also [2]); the hypergroup-like objects studied by Sunder [26]; the convolution algebras studied by Szwarc [28]; and the fusion rule algebras

Received October 11, 1994.