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KONTSEVICH'S INTEGRAL FOR THE KAUFFMAN POLYNOMIAL

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1. Introduction

Kontsevich's integral is a knot invariant which contains in itself all knot invariants of finite type, or Vassiliev's invariants. The value of this integral lies in an algebra $\mathscr{A}_0$, spanned by chord diagrams, subject to relations corresponding to the flatness of the Knizhnik-Zamolodchikov equation, or the so called infinitesimal pure braid relations [11].

For a Lie algebra $\mathfrak{g}$ with a bilinear invariant form and a representation $\rho : \mathfrak{g} \to \mathrm{End}(V)$ one can associate a linear mapping $W_{\mathfrak{g},t,\rho}$ from $\mathscr{A}_0$ to $\mathbf{C}[[h]]$, called the weight system of $\mathfrak{g}$, t, ρ. Here t is the invariant element in $\mathfrak{g} \otimes \mathfrak{g}$ corresponding to the bilinear form, i.e. $t = \sum_\alpha e_\alpha \otimes e_\alpha$ where $\{e_\alpha\}$ is an orthonormal basis of $\mathfrak{g}$ with respect to the bilinear form. Combining with Kontsevich's integral we get a knot invariant with values in $\mathbf{C}[[h]]$. The coefficient of h^n is a Vassiliev invariant of degree n.

On the other hand, for a simple Lie algebra $\mathfrak{g}$, $t \in \mathfrak{g} \otimes \mathfrak{g}$ as above, and a finite dimensional irreducible representation ρ, there is another knot invariant, constructed from the quantum R-matrix corresponding to $\mathfrak{g}$, t, ρ. Here R-matrix is the image of the universal quantum R-matrix lying in $\mathscr{U}_q(\mathfrak{g}) \otimes \mathscr{U}_q(\mathfrak{g})$ through the representation ρ. The construction is given, for example, in [17, 18, 20]. The invariant is a Laurent polynomial in q, by putting $q = \exp(h)$ we get a formal series in h. By a theorem of Bar-Natan, Birman and Lin the coefficient of h^n is a Vassiliev invariant of degree n, and its n-th derivative is the same as that of the invariant defined in the previous case. Bar-Natan conjectured that these two invariants are the same.

Kontsevich invented his integral by using ideas from Drinfeld's works on quasi-Hopf algebras [6, 7]. From these works it is also more or less clear that Kontsevich's integral, via weight system, should be the same as the invariant coming from quantum groups. But since Drinfeld's work does not treat knot invariant

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