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GLOBAL GENERATION OF ADJOINT BUNDLES

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1. Introduction

In 1988, I. Reider proved that for a smooth projective surface X and an ample line bundle L on X, $K_X + 3L$ is globally generated and $K_X + 4L$ is very ample ([12]). In fact his theorem is much stronger than this (see [12] for detail). Recently a lot of results have been obtained about effective base point freeness (cf. [1, 3, 8, 13, 14, 15]). In particular J. P. Demailly proved that $2K_X + 12n^nL$ is very ample for a smooth projective n-fold X and an ample line bundle L on X. [2] will give a good overview for these recent results. The motivation of these works is the following conjecture posed by T. Fujita.

CONJECTURE ([4]). *Let X be a smooth projective n-fold defined over $\mathbf{C}$ and let L be an ample line bundle on X. Then $K_X + (n+1)L$ is generated by global sections and $K_X + (n+2)L$ is very ample.*

We note that Fujita's conjecture is trivial if L is very ample by induction on $\dim X$. In the above situation, it is easy to see that $K_X + (n+1)L$ is nef and $K_X + (n+2)L$ is ample by using the theory of extremal rays (Mori theory cf. ([10, 6]). Moreover by using the base point free theorem ([7, p. 581, Theorem 6.1]), $K_X + (n+1)L$ is semiample, i.e. there exists a positive integer m such that $m(K_X + (n+1)L)$ is generated by global sections. The number $n+1$ is nothing but the maximal length of extremal rays of smooth projective n-folds. In this paper, we shall prove the following theorem.

THEOREM 1. *Let X be a smooth projective variety over $\mathbf{C}$ of dimension n and let L be an ample line bundle on X. Then $K_X + mL$ is generated by global sections on X for every*

$$m \geq n(n+1)/2 + 1.$$

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