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# ON $p$-ADIC DEDEKIND SUMS

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## §1. Introduction

For positive integers $h$, $k$ and $m$, the higher-order Dedekind sums are defined by

$$S_{m+1}^{(r)}(h, k) = \sum_{a=0}^{k-1} \bar{B}_{m+1-r}\left(\frac{a}{k}\right)\bar{B}_r\left(\frac{ha}{k}\right), \quad 0 \leq r \leq m+1,$$

where $\bar{B}_n(x)$, $n \geq 0$, are the Bernoulli functions (§2). If $m$ is odd and $(h, k) = 1$, the sum $S_{m+1}^{(m)}(h, k)$ is identical with the higher-order Dedekind sum of Apostol [1],

$$s_m(h, k) = \sum_{a=1}^{k-1} \frac{a}{k} \bar{B}_m\left(\frac{ha}{k}\right).$$

Recently, Rosen and Snyder [6] constructed a $p$-adic continuous function $S_p(s; h, k)$ for an odd prime $p$, which takes the values

$$S_p(m; h, k) = \begin{cases} k^m s_m(h, k) - p^{m-1}k^m s_m((p^{-1}h)_k, k), & \text{if } (k, p) = 1, \\ k^m s_m(h, k), & \text{if } k = p, \end{cases}$$

at positive integers $m$ such that $m + 1 \equiv 0 \pmod{p-1}$; here $(p^{-1}h)_k$ denotes the integer $x$ such that $0 \leq x < k$ and $px \equiv h \pmod{k}$.

The purpose of this paper is to extend this result of them to $k^m S_{m+1}^{(r)}(h, k)$ for every $h$, $k$ and $r \geq 1$. To this end, we use an expression of $k^m S_{m+1}^{(r)}(h, k)$ in terms of the Euler numbers ([2], [3]) and a $p$-adic continuous function which interpolates these numbers ([7], [8]).

Let $p$ be a prime number and $Z_p$ the ring of rational $p$-adic integers. Let $e = p-1$ or $e = 2$ according as $p > 2$ or $p = 2$. In §§2-3, we shall prove the following

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