

AN EXPLICIT FORMULA FOR THE FOURIER COEFFICIENTS OF SIEGEL-EISENSTEIN SERIES OF DEGREE 3

HIDENORI KATSURADA¹

Abstract. Using an induction formula of local densities by Kitaoka, we give an explicit formula for the Fourier coefficients of Siegel Eisenstein series of degree 3.

§1. Introduction

Let k be an even integer such that $k \geq n + 2$ and

$$E_k(Z) = \sum_{\{C,D\}} |CZ + D|^{-k}.$$

Siegel Eisenstein series of degree n and of weight k , where $\{C, D\}$ runs over all representatives of the equivalence classes of coprime symmetric pairs of degree n . Then $E_k(Z)$ has the following Fourier expansion:

$$E_k(Z) = \sum_C c_k(C) \exp(2\pi i \operatorname{Tr}(CZ)),$$

where C runs over all semi-positive definite half-integral matrices of degree n over \mathbf{Z} , and Tr denotes the trace.

The Fourier coefficient of Siegel-Eisenstein series is one of the most important subjects in number theory, and many contributions have been done to it. But we have no explicit formula for it except for a few cases. The case of degree 1 is well known. In [Ma1], [Ma2], Maaß gave an explicit formula for the case of degree 2. In [Ki2] Kitaoka essentially gave an explicit formula for $c_k(B)$ when $n = 3$ and B is \mathbf{Z}_2 -maximal using his recursion formula in [Ki1] (for the definition of \mathbf{Z}_2 -maximal see Section 2). Partial results for the case where B is not \mathbf{Z}_2 -maximal were given in [O-W].

Received September 20, 1995.

¹Partially supported by Grant-in-Aid for scientific Research, the Ministry of Education, Science, Sports and Culture, Japan