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ON RELATIVE BASE POINT FREENESS OF ADJOINT BUNDLE

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Abstract. We give an effective result on the relative base point freeness of an adjoint bundle for a pair of a projective morphism and a relatively ample line bundle.

§1. Introduction

Recently, Angehrn and Siu [AS] and Tsuji [Tj] independently obtained results on the following:

FUJITA'S FREENESS CONJECTURE OF ADJOINT BUNDLES. ([F]) *Let X be an n-dimensional projective manifold defined over $\mathbb{C}$ with an ample line bundle L. Then the adjoint bundle $\mathcal{O}_X(K_X \otimes L^{\otimes m})$ is generated by global sections for every $m > n$.*

Their effective bounds are $m > n(n+1)/2$. The basic ideas of their proofs from [AS] and [Tj] (use of Riemann-Roch theorem, Nadel's vanishing theorem, Ohsawa-Takegoshi's L^2-extension theorem and so on) are extremely simple and can be applied to a variety of contexts. In this note we would like to go into detail about the method and consider the following relative version:

MAIN THEOREM. *Let $f : X \longrightarrow Y$ be a projective morphism from a complex manifold X to a complex space Y, and let L be a relatively ample line bundle on X. Then $\mathcal{O}_X(K_X \otimes L^{\otimes m})$ is f-free, i.e., the natural sheaf homomorphism*

$$f^* f_* \mathcal{O}_X(K_X \otimes L^{\otimes m}) \longrightarrow \mathcal{O}_X(K_X \otimes L^{\otimes m}) \quad \textit{is surjective,}$$

for every

$$m > \frac{1}{2}\, d\,(d+1),$$

here d is the maximum dimension of the fibres of f.

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