

INTEGRATION OF LOCAL ACTIONS ON HOLOMORPHIC FIBER SPACES

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Abstract. It is proved that every holomorphically convex complex space endowed with an action of a compact Lie group K can be realized as an open K -stable subspace of a holomorphically convex space endowed with a holomorphic action of the complexified group $K^{\mathbb{C}}$. Similar results are obtained for holomorphic K -bundles over such spaces.

Let G be a real Lie group which acts by holomorphic transformations on a (reduced) complex space X . Suppose that the Lie algebra of the complexification $G^{\mathbb{C}}$ of G (see [Ho, p. 204]) is the complexification of the Lie algebra of G . This holds for example in the case where G is simply connected. Then, by integrating the holomorphic vector fields given by the G -action, the complexification $G^{\mathbb{C}}$ acts locally and holomorphically on X (see [K]).

Adapting the terminology of Palais (see [P]), we say that a complex space X^* which contains X as an open subset is a globalization of the complex G -space X whenever the local $G^{\mathbb{C}}$ -action on X extends to a global holomorphic action on X^* and $G^{\mathbb{C}} \cdot X = X^*$.

The following results are proved in this paper.

THEOREM 1. *Let G be a compact Lie group and X a holomorphically convex complex G -space X . Then there exists a globalization X^* of X satisfying the following conditions*

- (i) X^* is holomorphically convex
- (ii) Every G -equivariant holomorphic map ψ from X into a complex space

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