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VOLUME-PRESERVING GEODESIC SYMMETRIES ON FOUR-DIMENSIONAL HERMITIAN EINSTEIN SPACES

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Abstract. We prove that a four-dimensional Hermitian Einstein space is weakly $*$-Einsteinian and use this result to show that all geodesic symmetries are volume-preserving (up to sign) if and only if it is local symmetric.

§1. Introduction

Riemannian manifolds such that all (local) geodesic symmetries are volume-preserving (up to sign) or equivalently, are divergence-preserving, have been introduced in [5] and are called *D'Atri spaces* [27]. The first examples which are not locally symmetric were discovered in [4], [6]. These are the naturally reductive homogeneous spaces. Since then, many other classes of examples has been found and studied. The main classes are the following: Riemannian g.o. spaces (i.e., spaces such that every geodesic is an orbit of a one-parameter group of isometries), commutative spaces (i.e., homogeneous spaces whose algebra of all differential operators which are invariant under all isometries is commutative), generalized Heisenberg groups, harmonic spaces (in particular, the Damek-Ricci examples), weakly symmetric spaces, $\mathcal{SC}$-spaces (i.e., spaces such that the principal curvatures of small geodesic spheres have antipodal symmetry), probabilistic commutative spaces, $\mathcal{TC}$- and $\mathcal{C}_0$-spaces (see [1], [2] for more details). Of course, any manifold which is locally isometric to one of these examples is also a D'Atri space. We refer to [1], [2], [12], [26], [28] for more information and further references, in particular to the papers where these spaces have been introduced and to the extensive survey paper [13].

It is worthwhile to note that a D'Atri space is always analytic in normal coordinates [10], [22]. Moreover, their classification is completely known for dimensions smaller than four [12] but for higher dimensions the problem is completely open. Further, to our knowledge, an example which is not locally

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