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Nagoya Math. J.
Vol. 147 (1997), 193–211

THE GENUS OF CURVES ON THE THREE DIMENSIONAL QUADRIC

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Abstract. By means of an *ad hoc* modification of the so-called "Castelnuovo-Harris analysis" we derive an upper bound for the genus of integral curves on the three dimensional nonsingular quadric which lie on an integral surface of degree $2k$, as a function of k and the degree d of the curve. In order to obtain this we revisit the Uniform Position Principle to make its use computation-free. The curves which achieve this bound can be conveniently characterized.

Introduction

The objects of investigation of this paper are the following two connected problems. *What are the possible geometric genera of integral curves C of degree d lying on a nonsingular three dimensional quadric Q_3 in $\mathbb{P}^4$ and on an integral surface S of degree $2k$ contained in Q_3?* As it is shown in this paper the above genera are bounded above by a function of d and k. *What is the structure of the curves for which the genus is maximum with respect to k and d?*

The above problems are natural questions stemming from the analogue problems that one can state by replacing, in what above, Q_3 by $\mathbb{P}^3$ and $2k$ by k. These were answered completely in the paper [JH]. The paper [G-P] (and its refinement contained in [E-P]) deals with the very similar questions of (i) determining the biggest possible genus for curves of degree d in $\mathbb{P}^3$ which do not lie on a surface of degree less than k, or lie on a surface of degree k and of (ii) understanding the curves for which the genus is the maximum possible. Going back to the quadric body ([A-S], §6) gives an answer to the problem of determining the maximum possible genus for curves which lie on a surface of degree $2k$ under the assumption $d > 2k(k-1)$. To do so they use the technique of [G-P], coupled with the idea of considering only hyperplane sections which are tangent to the quadric Q_3.

In this work an upper bound for the above genera is worked out with no assumptions on the degree d. The bound is obtained pursuing some

Received August 31, 1994.