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SOBOLEV AND LIPSCHITZ ESTIMATES FOR WEIGHTED BERGMAN PROJECTIONS

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Dedicated to the memory of Professor Yau-Cheun Wong

Abstract. Let Ω be a bounded, decoupled pseudo-convex domain of finite type in $\mathbb{C}^n$ with smooth boundary. In this paper, we generalize results of Bonami-Grellier [BG] and Bonami-Chang-Grellier [BCG] to study weighted Bergman projections for weights which are a power of the distance to the boundary. We define a class of operators of Bergman type for which we develop a functional calculus. Then we may obtain Sobolev and Lipschitz estimates, both of isotropic and anisotropic type, for these projections.

§1. Introduction

Let $\Omega \subset \mathbb{C}^n$ be a bounded, smooth pseudo-convex domain. Then, Ω is said to be decoupled of finite type near $\zeta \in \partial\Omega$ if there exists a holomorphic coordinate system $(z_1, \ldots, z_n)$ mapping ζ onto 0 and a neighborhood U_ζ of ζ onto a neighborhood U of 0 and smooth, sub-harmonic but not harmonic functions $\{f_j\}_{\{j=1,\ldots,n-1\}}$, $f_j : \mathbb{C} \to \mathbb{R}$ with $f_j(0) = 0$, and each f_j vanishing to finite order at 0, such that

$$(1.1) \qquad \left\{z \in U : \rho(z) = 2\,\mathrm{Im}(z_n) - \sum_{j=1}^{n-1} f_j(z_j) > 0\right\} \simeq \Omega \cap U_\zeta.$$

Let us denote by $m_j(\zeta)$ the order of vanishing of f_j at 0.

Notice that the finiteness condition here is equivalent to finite type in the case of real analytic pseudo-convex hypersurface $\mathcal{Z} \subset \mathbb{C}^n$ since the Levi form of Ω is diagonalizable (see Kohn [K1], [K2]).

Let $\zeta \in \partial\Omega$. We denote by

$$(m_1, \ldots, m_{n-1}) = \max_{\zeta \in \partial\Omega}(m_1(\zeta), \ldots, m_{n-1}(\zeta)).$$

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