A. Fujii and Y. Kitaoka
Nagoya Math. J.
Vol. 147 (1997), 137–146

ON PLAIN LATTICE POINTS WHOSE COORDINATES ARE RECIPROCALS MODULO A PRIME

AKIO FUJII AND YOSHIYUKI KITAOKA

Abstract. We consider, for a given large prime p, the problem of covering a square $[0,p] \times [0,p]$ with discs center at the lattice point (x,y), x and y subject to condition $xy \equiv 1 \pmod{p}$ and with radius r. We are concerned with the size of r.

§1. Introduction

In this paper we consider, for each given large prime P, the problem of covering a 2-dimensional box $[0,P] \times [0,P]$ with discs $C_{(x,y)}(r)$ center at the lattice point (x,y), x and y subject to the condition $xy \equiv 1 \pmod{P}$ and with the least possible radius r. In other words, we wish to determine the infimum $r(P)$ of r satisfying

$$\bigcup_{\substack{x=1 \\ xy\equiv 1 \pmod{P}}}^{P-1} C_{(x,y)}(r) \supset [0,P] \times [0,P].$$

When $r = \sqrt{P}$, the area of the left-hand side member is roughly P^2, even if discs do not overlap. Thus it may be too optimistic to expect to have

$$r(P) = \text{constant times } \sqrt{P},$$

and actually for $P = 5$, $r = \sqrt{5}$ is not large enough, but it would be reasonable to conjecture that

$$r(P) = P^{\frac{1}{2}+\varepsilon}$$

for every $\varepsilon > 0$. If this is the case, we may claim that the lattice points (x,y) with $xy \equiv 1 \pmod{P}$ are "uniformly distributed." Towards this conjecture, we shall prove the following theorem.

Received July 14, 1995.