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ON THE STABILITY OF PSEUDOCONVEXITY FOR CERTAIN COVERING SPACES

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Abstract. It is proved that, if $X \overset{\pi}{\to} T$ is a proper holomorphic map with one-dimensional fibers and $\tilde{X} \overset{\varpi}{\to} X$ a covering map, a point $t \in T$ has a neighbourhood U such that $\varpi^{-1}(\pi^{-1}(U))$ is holomorphically convex if and only if $\varpi^{-1}(\pi^{-1}(t))$ is holomorphically convex.

§1. It has long been known that the parameter space of a complex analytic family of complex manifolds often admits a significant geometric structure that deserves intensive studies. To describe such a structure, theory is extremely useful when the fibers are Riemann surfaces, because any family is then transformed into that of quasifuchsian groups in PSL$(2, \mathbf{C})$. In higher dimensions, however, the uniformization theory has not been developed enough to capture important aspects of the variation of complex structures. Nevertheless it seems natural to expect that covering spaces somehow carry information about the deformation. Thus we would like to continue the previous work [O-2] on the holomorphic convexity of the covering spaces of a family of compact Riemann surfaces over the unit disc.

§2. First we ask for the possibility of constructing a plurisubharmonic function on the covering of the family by extending one from the special fiber. We shall give an answer to this question which is almost trivial (see Theorem 1). However, as we shall see after Theorem 2, this poses an interesting question on the extension of bounded plurisubharmonic functions. Next we would like to extend the result in [O-2] to the case of families with singular fibers. It will turn out that the absence of infinite chain of compact complex curves is sufficient for the holomorphic convexity at the germ level (cf. Theorem 3).

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