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SEMI-CLASSICAL BOUNDS ON SCATTERING CROSS SECTIONS IN TWO DIMENSIONAL MAGNETIC FIELDS

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Abstract. We prove the uniform boundedness of averaged total cross sections or of quantities related to scattering into cones in the semi-classical limit for scattering by two dimensional magnetic fields. We do not necessarily assume that the energy under consideration is in a non-trapping energy range in the sense of classical dynamics.

§1. Introduction

The present work is a continuation to [20] where we have studied the shadow scattering (the quantum total cross section doubles the classical one in the semi-classical limit) in magnetic fields under the assumption that the energy under consideration is in a non-trapping energy range in the sense of classical dynamics and we have proved that the shadow scattering is in gerenal violated in the case of scattering by magnetic fields. We here study the problem about the uniform boundedness of averaged total cross sections or of quantities related to scattering into cones in the semi-classical limit without assuming such a non-trapping energy condition. In final section (Section 9), we also study the bound on cross sections for scattering by magnetic fields with small support. As a conclusion, we can obtain that such a bound seems to depend on the flux of magnetic fields.

Throughout the whole exposition, we work exclusively in the two dimensional space R^2 with generic point $x = (x_1, x_2)$. Let $A(x) = (a_1(x), a_2(x)) : R^2 \to R^2$ be a smooth magnetic vector potential and let

$$H(A) = (-i\nabla - A)^2/2 = \sum_{j=1}^{2}(D_j - a_j)^2/2 \tag{1.1}$$

be the Schrödinger operator associated with magnetic potential A, where $D_j = -i\partial_j = -i\partial/\partial x_j$. We sometimes identify A with the one-form $A =$

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