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STABILITY OF HÖLDER ESTIMATES FOR $\bar\partial$ ON PSEUDOCONVEX DOMAINS OF FINITE TYPE IN $\mathbb{C}^2$

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Abstract. Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^2$ and let $b\Omega$ be of finite type m. Then we prove the stability of Hölder estimates for $\bar\partial$ under some perturbations of $b\Omega$. As an application, we prove the Mergelyan property with respect to $C^\alpha(\overline{\Omega})$ norms for $0 \le \alpha < 1/m$.

§1. Introduction

Methods of integral representations for estimating solutions for $\bar\partial$-equation in several complex variables have been successfully used for strongly pseudoconvex domains [G-L, H, R1]. For weakly pseudoconvex domains of finite type in $\mathbb{C}^2$, Range [R2] has introduced a method for constructing integral kernels on smoothly bounded pseudoconvex domains. This method was based on Skoda's L^2 estimates [S] for holomorphic solutions $h_j(p,z)$, $j = 1, 2$, of the division problem

$$h_1(p,z)(z_1 - p_1) + h_2(p,z)(z_2 - p_2) = 1, \quad p \in b\Omega, \ z \in \Omega.$$

He has used the detailed geometric analysis of Catlin [C], near a boundary point $p_0 \in b\Omega$ of finite type to get pointwise estimates of $h_j(p,z)$, $z \in \Omega$, $j = 1, 2$. The result was:

THEOREM. ([R2]) *Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^2$ of finite type m, and let $f \in C^1_{0,1}(\overline{\Omega})$ be $\bar\partial$-closed. Then for every $\eta > 0$, there is a solution $u^{(\eta)}$ of $\bar\partial u = f$ on Ω which satisfies*

$$|u^{(\eta)}(z) - u^{(\eta)}(w)| \le C_\eta \|f\|_{L^\infty(\Omega)} |z - w|^{\frac{1}{m} - \eta} \tag{1.1}$$

for $z, w \in \Omega$. The constant C_η is independent of f.

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