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HYPERCONVEXITY AND BERGMAN COMPLETENESS

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Abstract. We show that any bounded hyperconvex domain is Bergman complete.

Let $D \subset \mathbb{C}^n$ be a bounded domain. By b_D we denote the Bergman distance on D which is defined as the integrated form of the Bergman metric

$$\beta_D(z;X) := \sqrt{\sum_{i,j=1}^{n} \frac{\partial^2 \log K_D(z,z)}{\partial z_i \partial \bar{z}_j} X_i \bar{X}_j},$$

i.e. $b_D(z',z'') := (\int \beta_D)(z',z'')$, $z',z'' \in D$, where $K_D(\cdot,\cdot)$ is the Bergman kernel of D (for more details see [9, Chapter IV]).

It is an old problem asked by Kobayashi (cf. [11], see also [12]) which bounded pseudoconvex domain $D \subset \mathbb{C}^n$ is Bergman complete. Observe that pseudoconvexity is necessary. There is a long list of papers treating this question (cf. [5], [7], [10], [13], [14], [15], [16]). The state of affair is that the Bergman kernel K_D tends to infinity near the boundary if D is hyperconvex (cf. [14]). Recall that a bounded domain D is called to be *hyperconvex* if there is a continuous negative plurisubharmonic exhaustion function. Observe that D is already hyperconvex if a negative (not necessarily continuous) plurisubharmonic exhaustion function of D exists (cf. [17], [3]). Using a result of P. Pflug (cf. [16]) density of $H^\infty(D)$ in $L^2_h(D)$ would imply that D is Bergman complete. Following this line Chen (cf. [5]) proved recently that any bounded pseudoconvex domain with Lipschitz boundary is b-complete. Observe that such domain is automatically hyperconvex (cf. [6]). In his paper, Chen asks the question whether any bounded hyperconvex domain is Bergman complete. In fact, his paper itself contains the key to solve that question in the affirmative. Namely, the following lemma is there.

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