H. Fujimoto
Nagoya Math. J.
Vol. 152 (1998), 131–152

UNIQUENESS PROBLEM WITH TRUNCATED MULTIPLICITIES IN VALUE DISTRIBUTION THEORY

HIROTAKA FUJIMOTO

Abstract. In 1929, H. Cartan declared that there are at most two meromorphic functions on $\mathbb{C}$ which share four values without multiplicities, which is incorrect but affirmative if they share four values counted with multiplicities truncated by two. In this paper, we generalize such a restricted H. Cartan's declaration to the case of maps into $P^N(\mathbb{C})$. We show that there are at most two nondegenerate meromorphic maps of $\mathbb{C}^n$ into $P^N(\mathbb{C})$ which share $3N+1$ hyperplanes in general position counted with multiplicities truncated by two. We also give some degeneracy theorems of meromorphic maps into $P^N(\mathbb{C})$ and discuss some other related subjects.

§1. Introduction

In 1926, R. Nevanlinna showed that, for two distinct nonconstant meromorphic functions f and g on the complex plane $\mathbb{C}$, they cannot have the same inverse images for five distinct values, and g is a special type of linear fractional transformation of f if they have the same inverse images counted with multiplicities for four distinct values([10]). In [2] $\sim$ [4], the author gave several types of generalizations of these theorems to the case of meromorphic maps of $\mathbb{C}^n$ into $P^N(\mathbb{C})$. He considered two distinct meromorphic maps f and g satisfying the condition that $\nu(f, H_j) = \nu(g, H_j)$ for q hyperplanes $H_1, H_2, \dots, H_q$ in $P^N(\mathbb{C})$ located in general position, where $\nu(f, H_j)$ means the map of $\mathbb{C}^n$ into $\mathbb{Z}$ whose value $\nu(f, H_j)(a)$ $(a \in \mathbb{C}^n)$ is the intersection multiplicity of the images of f and H_j at $f(a)$. He proved that $q \leq 3N+1$ if either f or g is (linearly) nondegenerate, and $q \leq 2N+2$ if either f or g is algebraically nondegenerate.

It is reasonable to ask whether these results remain valid regardless of multiplicity or not. There are several results without multiplicities in some restricted situations. To state some of them, we take a nondegenerate meromorphic map g of $\mathbb{C}^n$ into $P^N(\mathbb{C})$, a positive integer ℓ_0 and q hyperplanes

Received June 26, 1997.