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ON STANDARD L-FUNCTIONS ATTACHED TO AUTOMORPHIC FORMS ON DEFINITE ORTHOGONAL GROUPS

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Abstract. We show an explicit functional equation of the standard L-function associated with an automorphic form on a definite orthogonal group over a totally real algebraic number field. This is a completion and a generalization of our previous paper, in which we constructed standard L-functions by using Rankin-Selberg convolution and the theory of Shintani functions under certain technical conditions. In this article we remove these conditions. Furthermore we show that the L-function of f has a pole at $s = m/2$ if and only if f is a constant function.

Introduction

The purpose of this paper is to prove a meromorphic continuation and a functional equation of the standard L-function attached to an auotomorphic form on a definite orthogonal group. In our previous paper [4], we have proposed an approach to construct standard L-functions associated with automorphic forms on classical groups. In particular, we proved an explicit functional equation of the standard L-function in the case of definite orthogonal groups over $\mathbf{Q}$ under certain conditions. In this paper, removing those technical conditions, we obtain a satisfactory result for the functional equation of the standard L-function.

To be more precise, let k be a totally real algebraic number field with maximal order $\mathfrak{o}_k$. Let $S \in M_m(\mathfrak{o}_k)$ be an even integral (totally) positive definite symmetric matrix of rank $m \geq 2$ and assume that $\mathfrak{o}_k^m$ is a maximal $\mathfrak{o}_k$-integral lattice with respect to S. We denote by G the orthogonal group of S. For each nonarchimedean place $\mathfrak{p}$, let $K_\mathfrak{p}^* = \{g \in G_\mathfrak{p} \mid (g-1)S^{-1} \in M_m(\mathfrak{o}_{k,\mathfrak{p}})\}$, where $G_\mathfrak{p}$ is the $\mathfrak{p}$-adic completion of G. Clearly $K_\mathfrak{p}^*$ is a normal subgroup of a maximal open compact subgroup $K_\mathfrak{p} = G_\mathfrak{p} \cap GL_m(\mathfrak{o}_{k,\mathfrak{p}})$. We consider the space $\mathfrak{S}(K_f^*)$ of left G_k and right $G_\infty \prod_{\mathfrak{p}<\infty} K_\mathfrak{p}^*$ invariant functions on the adelized group G_A of G, where G_∞ means the direct product of

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