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TENSOR PRODUCTS OF MATRIX FACTORIZATIONS

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To the memory of Professor Hideyuki Matsumura

Abstract. Let K be a field and let $f \in K[[x_1, x_2, \dots, x_r]]$ and $g \in K[[y_1, y_2, \dots, y_s]]$ be non-zero and non-invertible elements. If X (resp. Y) is a matrix factorization of f (resp. g), then we can construct the matrix factorization $X \mathbin{\widehat{\otimes}} Y$ of $f + g$ over $K[[x_1, x_2, \dots, x_r, y_1, y_2, \dots, y_s]]$, which we call the tensor product of X and Y.

After showing several general properties of tensor products, we will prove theorems which give bounds for the number of indecomposable components in the direct decomposition of $X \mathbin{\widehat{\otimes}} Y$.

§0. Introduction

Let K be a field and let $K[[x]]$ be the formal power series ring in the variables $x = x_1, x_2, \dots, x_r$ and $K[[y]]$ the formal power series ring in $y = y_1, y_2, \dots, y_s$. And let $f \in K[[x]]$ and $g \in K[[y]]$ be non-zero and non-invertible elements. We consider the problem how one can relate MCM modules (= maximal Cohen-Macaulay modules) over $R_1 = K[[x]]/(f)$ and over $R_2 = K[[y]]/(g)$ with MCM modules over $R = K[[x, y]]/(f + g)$. Actually if $R_2 = K[[y_1]]/(y_1^2)$ or if $R_2 = K[[y_1, y_2]]/(y_1 y_2)$, then the problem was considered by Knörrer [K1], [K2], and a certain periodicity theorem for MCM modules is known in this course. (cf. [K2, Theorem 3.1].) On the other hand, Herzog and Popescu [HP] had considered the same problem for the particular case $R_2 = K[[y_1]]/(y_1^3)$ and they call the problem "Thom-Sebastiani problem".

In this paper, we will present a method of tensor products of matrix factorizations to relate those objects. More precisely, if we have two MCM modules M and N respectively on the rings R_1 and R_2, we can construct a new MCM module $M \mathbin{\widehat{\otimes}} N$ over R by means of tensor product. In such a construction, it may happen that even if M and N are indecomposable, the tensor product $M \mathbin{\widehat{\otimes}} N$ might be decomposable. Thus it will be a

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