A GROUP OF AUTOMORPHISMS OF THE HOMOTOPY GROUPS

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It is well known that the fundamental group $\pi_1(X)$ of an arcwise connected topological space X operates on the n-th homotopy group $\pi_n(X)$ of X as a group of automorphisms. In this paper I intend to construct geometrically a group $\mathfrak{A}(X)$ of automorphisms of $\pi_n(X)$, for every integer $n \geqq 1$, which includes a normal subgroup isomorphic to $\pi_1(X)$, so that the factor group of $\mathfrak{A}(X)$ by $\pi_1(X)$ is completely determined by some invariant $\Sigma(X)$ of the space X. The complete analysis of the operation of the group on $\pi_n(X)$ is given in § 3, § 4, and § 5.

Throughout the whole paper, X denotes an arcwise connected topological space which has such suitable homotopy extension properties as a polyhedron does, and all mappings are continuous transformations.

§ 1. Definition of the group $\mathfrak{A}(X)$.

Let x_0 be an arbitrary point of the space X, and Ω a collection $X^X(x_0, x_0)$ of all the mappings that transform X into X and x_0 into x_0. For two maps $a, b \in \Omega$, a is said to be homotopic to b (in notation : $a \sim b$) if there exists a homotopy $h_t \in \Omega$ (for $1 \geqq t \geqq 0$) such that $h_0 = a$ and $h_1 = b$. A mapping $a \in \Omega$ is called to have a (two sided) homotopy inverse, if there is a map $\varphi \in \Omega$ such that $a\varphi \sim 1$ and $\varphi a \sim 1$, where 1 denotes the identity transformation of X onto itself. Let Ω^* be the collection of all the mappings belonging to Ω, each of which has a homotopy inverse.

Now let $X \times I$ be the topological product of X and the line segment I between 0 and 1, and let us consider the totality U of the mappings $\theta : X \times I \to X$ which satisfy the following conditions :

$$\text{(1.1)} \qquad \left.\begin{array}{ll} \text{i)} & \theta \mid X \times 0 \in \Omega^* \\ \text{ii)} & \theta(x_0, 1) = x_0 \end{array}\right\}$$

For two maps $\theta, \theta' \in U$, θ is homotopic to θ' (notation : $\theta \sim \theta'$) if there exists a homotopy $h_t : X \times I \to X$ (for $1 \geqq t \geqq 0$) such that

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