AN EXTENSION OF POINCARÉ FORMULA IN INTEGRAL GEOMETRY

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1. A curve c_2 of finite length L_2 moves on a euclidean plane. Let the number of points of intersection of c_2 with the fixed curve c_1 of length L_1 be n, and the element of kinematic measure of the position of c_2 be dK. Then, owing to Poincaré, we have

$$\int n\, dK = 4L_1L_2,$$

where the integration extends over all the positions of the moving curve c_2. An analogous formula was obtained by Santaló [1] in the case of a curve and a surface in the euclidean 3-space, and by Blaschke [2] in the case of two surfaces. Here I extend these to the case of general Klein spaces by the method of moving frames of E. Cartan [3]. The method used is analogous to that of the paper of S. S. Chern [4], but I have worked out independently. Moreover I show examples which may be of some interest.

2. In Kein spaces, whose fundamental group is a Lie group G, we call the left cosets aH of G by a Lie subgroup H points, and let F_1 and F_2 be manifolds which consist of points x, the former being space fixed and the latter moving. Hereafter we assume the differentiability to the order we need. We attach to every point of F_1 and F_2 Frenet's frames, whose motion along F_1 and F_2 is denoted by S_1 and S_2. Then we take one of the intersection points and call it O. Let the motion, which removes the Frenet's frame of F_1 at O to the one of F_2 at O, be T and let the fixed frame of F_1 be R_0 and the frame that is relatively fixed to F_2 be R. Then R can be represented as

$$R = S_1 T S_2^{-1} R_0,$$

which can be understood by the fact that the relative position between R and $S_1 T R_0$ is represented by S_2^{-1}. So when we put

$$S = S_1 T S_2^{-1}, \tag{1}$$

the position of moving manifold F_2 can be determined by S. We denote the

Received Sept. 20, 1950.