ON SOME PROPERTIES OF LOCALLY COMPACT GROUPS WITH NO SMALL SUBGROUP

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1. Let G be a locally compact group. Under a neighbourhood U we mean a symmetric (i.e. $U=U^{-1}$) neighbourhood of the identity e, with the compact closure $\overline{U}$. If there exists a neighbourhood U containing no subgroup other than the identity group, we say that G has *no small subgroup*. Now G has been called to have *property* (S) if

(S) for every $x \neq e$ in a sufficiently small neighbourhood U there exists an integer n so that $x^{2^n} \notin U$.[1]

If G has property (S), G is obviously with no small subgroup. Conversely we have

THEOREM 1. *A locally compact group has property* (S) *if it has no small subgroup.*

Proof. Let G be a locally compact group and V a neighbourhood with closure having no subgroup other than the identity group. Let W be a neighbourhood such as $W^2 \subset V$.

Suppose that the theorem is not true. Then there exist sequences $\{U_n\}$ and $\{x_n\}$ of neighbourhoods and elements such that

$$\ldots \supset U_n \supset U_{n+1} \supset \ldots,$$
$$\cap\, U_n = e,$$
$$U_n \ni x_n^{2^m}, \quad x_n \neq e, \quad m = 0, 1, 2, \ldots .$$

Because $\overline{V}$ has no non-trivial subgroup there exists j_n such that

$$x_n \in W, \ \ldots, \ x_n^{j_n - 1} \in W, \ x_n^{j_n} \notin W,$$

for every n. Then the inequality $2^{m_n - 1} < j_n \leqq 2^{m_n}$ determines a unique integer m_n. It is to be remarked that if $1 \leqq s_n \leqq 2^{m_n}$, then $x_n^{s_n} \in W^2 \subset V$. In particular $x_n^{j_n}$ is contained in V. Hence we can choose a subsequence $\{x_{n'}\}$ of $\{x_n\}$ such that $\lim x_{n'}^{j_{n'}}$ exists. Then the fact that $x_{n'}^{j_{n'}} \notin W$ implies that

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[1] See Kuranishi [4].