SOME STUDIES ON GROUP CHARACTERS

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INTRODUCTION

The theory of group characters was originated by G. Frobenius and has been studied by many authors including, above all, I. Schur and W. Burnside. As to the modular theory we owe it to recent works by R. Brauer and his collaborators, including T. Nakayama and C. Nesbitt, which clarified also the connections between the structure theory and the representation theory.

By using these results, especially those of R. Brauer we want to discuss in the present paper the correspondence between the blocks of characters of G and those of N, where N is a normal maximal subgroup of G (§ 2). Further we will prove several propositions on soluble groups which connect the structure theory with the representation theory to a certain degree and are proved by the arguments of § 2 (§ 3).

§ 1. **Preliminaries**

Let us first cite some known results, mostly due to R. Brauer and C. Nesbitt. Let G be a group of finite order g and let K be the field of the primitive g-th root of unity over the rational number field. Then all the irreducible representations of G are realizable in K. Let $K_1, K_2, \ldots, K_n$ be the classes of conjugate elements in G and let $\chi_1, \chi_2, \ldots, \chi_n$ be the irreducible characters of G. Let p be a rational prime number and let $\mathfrak{p}$ be a prime divisor of p in K. Then we say that χ_i and χ_j belong to the same block (for $\mathfrak{p}$) if and only if we have $g_\nu\chi_i(K_\nu)/z_i \equiv g_\nu\chi_j(K_\nu)/z_j \pmod{\mathfrak{p}}$ for $\nu = 1, 2, \ldots, n$, where z_i and z_j are the degrees of χ_i and χ_j respectively, and g_ν is the index of the centralizer of the element in K_ν in G, and $\chi_i(K_\nu)$ is the value which χ_i takes at K_ν. Let $g = p^a g'$, where $(p, g') = 1$. If in a block all the degrees of characters belonging to it are divisible by p^α and at least one of them is not divisible by $p^{\alpha+1}$, then the block is said to be of defect $a-\alpha$. On the other hand if ρ_ν is the maximal exponent of p dividing g/g_ν then we call K_ν a class of defect ρ_ν. Further K_ν is called p-regular if, and only if, it contains an element which has an order prime to p.

The following four papers will often be quoted and refered to as BN_1, BN_2, B_1 and B_2, respectively.

R. Brauer and C. Nesbitt, On the modular representations of groups of finite

Received July 19, 1950.