CORRECTIONS AND SUPPLEMENTARIES TO MY PAPER CONCERNING KRULL-REMAK-SCHMIDT'S THEOREM

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1. It has recently been found that my previous paper "On generalized semi-primary rings and Krull-Remak-Schmidt's theorem," Jap. Journ. Math. **19** (1949) — referred to as S. K. — contained in its Theorems 19 and 20 some errors. Nevertheless the writer has been able to correct them in suitable forms so that most parts of both theorems hold, even under a weaker assumption, and also subsequent theorems remain valid. These will be, together with some supplementary remarks, shown in the present note.[1]

For completeness let us recall several definitions. Let R be any (associative) ring. An element c of R is called a *root element* if there exists no non-zero element x such that $xax = x$, or what comes to the same, if the left ideal Rc, or equivalently, the right ideal cR contains no non-zero idempotent element. We denote by C the set of all root elements of R. Then in order that C forms a two-sided ideal it is sufficient that C is additively closed, that is, the sum of any two root elements is also root element. And, when this is the case, we say that R possesses the *radical* C. R is called *semi-primary* if R possesses a radical (not identical with R) and every non-zero idempotent element contains a primitive idempotent element; if moreover all primitive idempotent elements are isomorphic to each other we call R *primary*. R is said to be *completely primary* when R possesses a radical (again not identical with R) and every non-zero idempotent element is primitive.

Now *suppose that every non-zero idempotent element in R contains a primitive idempotent element. Then R possesses a radical* (*i.e.* R *is semi-primary*) *if and only if for every primitive idempotent element e the subring eRe possesses a radical* (*i.e.* eRe *is completely primary*). For the proof we have only to prove the "if" part, since $eCe = eRe \cap C$ is the set of all root elements of eRe,[2] and we may assume that eRe possesses the radical eCe. Suppose that there were two root elements a, b such that $a + b$ is not a root element (of R). Then $R(a + b)$

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[1] We take this opportunity to correct the following errata: in the sixth line following the proof of Theorem 16 (page 537) both $\mathfrak{R}$ should be replaced by $\mathfrak{N}$.

[2] S. K. Lemma 6.