Nihonkai Math. J.
Vol.8 (1997), 179-194

ON THE CLASS $\mathcal{Y}$ OPERATORS

Atsushi UCHIYAMA and Takashi YOSHINO

Abstract. In [9], one of the authors proved that, for a M-hyponormal operator A^* and for a dominant operator B, $CA = BC$ implies $CA^* = B^*C$. In the case where A^* and B are normal, this result are well known as the Putnam-Fuglede theorem. In this paper, we will generalize this result to the cases where A^* or both A^* and B belong to some class $\mathcal{Y}$ which includes the class of M-hyponormal operators. And also we prove that every compact operators in the class $\mathcal{Y}$ are normal.

We denote the set of all bounded linear operators on a Hilbert space $\mathcal{H}$ by $\mathcal{B}(\mathcal{H})$. The following results are well known.

Fuglede's theorem. ([2]) If $T \in \mathcal{B}(\mathcal{H})$ is normal and if $TS = ST$ for some $S \in \mathcal{B}(\mathcal{H})$, then $T^*S = ST^*$.

Putnam's corollary. ([3]) If A^* and B are normal operators on Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$ respectively and if C is a bounded linear operator from $\mathcal{H}$ to $\mathcal{K}$ such that $CA = BC$, then $CA^* = B^*C$.

Lemma 1. ([1]) For $A,\ B \in \mathcal{B}(\mathcal{H})$, the following assertions are equivalent.

(i) $A\mathcal{H} \subseteq B\mathcal{H}$.

(ii) $AA^* \leq \lambda^2 BB^*$ for some $\lambda \geq 0$.

(iii) There exists a $C \in \mathcal{B}(\mathcal{H})$ such that $A = BC$.

In particular, there exists a $C \in \mathcal{B}(\mathcal{H})$ uniquely such that

$$(a)\ \ \|C\|^2 = \inf\{\mu\ ;\ AA^* \leq \mu BB^*\}$$

$$(b)\ \ \mathcal{N}_A = \mathcal{N}_C \qquad \text{and} \qquad (c)\ \ C\mathcal{H} \subseteq [B^*\mathcal{H}]^{\sim}.$$

1991 *Mathematics Subject Classification* : 47B20.