

TWO CONDITIONS ON THE RICCI TENSOR OF A REAL HYPERSURFACE OF COMPLEX PROJECTIVE SPACE

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ABSTRACT. We study two conditions on the Ricci tensor of a real hypersurface of complex projective space that allows us to characterize certain real hypersurfaces. We also introduce a new kind of such real hypersurfaces.

§1. Introduction

Let CP^m be a complex projective space of complex dimension m endowed with the Fubini-Study metric g of constant holomorphic sectional curvature 4. Let M be a connected real hypersurface of CP^m and N a local unit normal vector field on M . Then $\xi = -JN$ is tangent to M , where J denotes the almost complex structure of CP^m .

Let us denote by A , R and S the shape operator, the curvature tensor and the Ricci tensor of M , respectively. We put $h = \text{trace} A$ and $H = hA - A^2$.

A real hypersurface M of CP^m is called *pseudo-Einstein* if its Ricci tensor satisfies

$$(1.1) \quad SX = aX + bg(X, \xi)\xi$$

for any vector field X tangent to M and some functions a, b on M , where g denotes the induced Riemannian metric on M . Pseudo-Einstein real hypersurfaces of CP^m are classified by the following theorem:

Theorem A. ([1]) *Let M be a complete pseudo-Einstein real hypersurface of CP^m , $m \geq 3$. Then M is locally congruent to one of the following spaces:*

- a) a geodesic hypersphere,
- b) a tube of radius r over a totally geodesic CP^k ($1 \leq k \leq m-2$), where

$$0 < r < \frac{\pi}{2} \text{ and } \cot^2 r = \frac{k}{m-k-1},$$

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