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ON SOME CIRCLES IN PSEUDO-RIEMANNIAN MANIFOLDS

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§1. Introduction.

Let $\widetilde{M}$ be a Riemannian manifold. A totally umbilical submanifold M of $\widetilde{M}$ with parallel mean curvature vector field is said to be an *extrinsic sphere* [2][1)].

One-dimensional extrinsic spheres are the curves c to be called *circles*, which were considered under the name of geodesic circles or curvature circles characterized by the following differential equations

$$\nabla_X \nabla_X X + \langle \nabla_X X, \nabla_X X\rangle X = 0,$$

where $\langle , \rangle$ is the metric, ∇ is covariant differentiation along c and X is the unit tangent vector field of c. For a circle c, $k := \langle \nabla_X X, \nabla_X X\rangle^{\frac{1}{2}}$ is a non-negative constant which is called the *curvature* of c. Especially $k = 0$, a circle c is a *geodesic*. The following theorems are well-known:

Theorem A([2]). *Let M (dim $M \geq 2$) be a connected Riemannian submanifold of a Riemannian manifold $\widetilde{M}$. For some $k > 0$, the following conditions are equivalent:*
(1) Every circle of radius k in M is a circle in $\widetilde{M}$,
(2) M is an extrinsic sphere in $\widetilde{M}$.

On the other hand, if the development of $c(s)$ in the tangent Möbius space is a circle, then $c(s)$ is called a *conformal circle* (cf. [1], [3]). Then the equation of the conformal circle is given by

$$\nabla_X \nabla_X X + \left(\langle \nabla_X X, \nabla_X X\rangle + \frac{1}{n-2}\langle SX, X\rangle\right) X - \frac{1}{n-2} SX = 0, \tag{1.1}$$

where S is the Ricci operator of M (dim $M = n \geq 3$). Remark that (1.1) is represented by the Riemannian metric and the Riemannian connection. Also they showed in [1] that, when every circle in M is a conformal circle in $\widetilde{M}$, M is totally umbilical in $\widetilde{M}$.

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1) Numbers in brackets refer to the references at the end of the paper.

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