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Bellman equations for discrete time two–parameter optimal stopping problems

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Abstract

We study Bellman equations associated with two-parameter optimal stopping problems for discrete time bi–Markov processes. The existence and the uniqueness of a solution of the Bellman equation for our problem are investigated by using the concept of the bi–excessive function.

Keywords : Bellman equation * bi–excessive function * bi–Markov process * strategy * tactic * two–parameter optimal stopping problem

1 Introduction

Throughout this paper we consider the stochastic processes indexed by $\mathbf{N}^2$. Let $\mathbf{T} = \mathbf{N}^2$. The index set $\mathbf{T}$ is extended to its one–point compactification $\mathbf{T} \cup \{\infty\}$ endowed with the following partial order : for all $z = (s,t), z' = (s',t') \in \mathbf{T}$,

$$\begin{array}{lll} z \leq z' & \text{if and only if} & s \leq s', t \leq t', \\ z < z' & \text{if and only if} & s < s', t < t', \\ z \leq \infty & \text{for all } z \in \mathbf{T}. & \end{array}$$

For $i = 1,2$, let $X^i = (\Omega^i, \mathcal{F}^i, \mathcal{F}^i_t, X^i(t), P^i_x)$ be a time homogeneous Markov chain with a state space $(E^i, \mathcal{B}^i)$. We assume that X^1 and X^2 are mutually independent.

We define a bi–Markov process introduced in Mazziotto [8] , that is, the family of a two–parameter process taking values in $E = E^1 \times E^2$

$$X(z) = (X^1(s), X^2(t)) \quad z = (s,t) \in \mathbf{T}$$

on the probability space $(\Omega = \Omega^1 \times \Omega^2, \mathcal{F} = \mathcal{F}^1 \otimes \mathcal{F}^2, P_{(x,y)} = P^1_x \otimes P^2_y, (x,y) \in E)$ endowed with the smallest two–parameter filtration $\{\mathcal{F}_z, z \in \mathbf{T}\}$ which contains $\{\mathcal{F}^1_s \otimes \mathcal{F}^2_t, (s,t) \in \mathbf{T}\}$ and satisfies the conditions