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A Note On An Inverse Parabolic Problem

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1. Introduction.

Let us consider the following Cauchy problem :

$$\partial_t u(x,t) = \Delta u(x,t) + q(x)u(x,t) \quad \text{in } \mathbf{R}^n \times (0,\infty)\,(n \geq 2), \tag{1.1}$$

$$u(x,0) = f(x) \quad \text{on } \mathbf{R}^n, \tag{1.2}$$

where $q(x)$, $f(x)$ are bounded continuous functions and $\operatorname{supp} q \subset\subset \{x : |x| < R\}\,(R > 0)$. Without loss of generality, we may assume $0 \notin \operatorname{supp} q$. Various inverse problems are studied for determining $q(x)$ from the additional infomations (cf. [2], [5]).

In this paper, we study the following inverse problem:
Determine $q(x)$ from the knowledge of $\{u(f)(R\omega,t) : \omega \in \mathbf{S}^{n-1}\}$ (considred as the set of observed data) and $\{f(x)\}$ (considered as the set of input data).

For the wave equation $u_{tt} = \Delta u + q(x)u$, their high frequency beam solutions had used to derive the uniqueness of $q(x)$ from the Neumann to Dirichlet map (cf. [4], [7], [9]). The Neumann to Dirichlet map uniquely determines the X-ray transformation of $q(x)$. However the parabolic equations can not have the beam type solutions. For the parabolic equation $u_t = \Delta u + q(x)u$, Theorem 9.1.2 in [5] shows that the maximum principle and the enery estimates for the parabolic one derive the uniqueness of $q(x)$ from the Neumann to Dirichlet map. Therefore we need another idea to obtain the X-ray transformation of $q(x)$. In the case of parabolic equations, by combining the Feynman-Kac formula and the n-dimensional Brownian bridge process, we can represent their solutions directly and we shall see that we can get the X-ray transformation of $q(x)$. These considerations leads us to the proof