Nihonkai Math. J.
Vol.14(2003), 99-112

A Characterization of Stability of Discrete Hopfield Neural Networks

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Abstract

We discuss the stability of discrete Hopfield neural networks (Shortly, DHNNs) in synchronous mode. To do this, we introduce an equivalent relation in the set $M_{m,n}(\mathbb{R})$ of all m by n matrices over the real field $\mathbb{R}$ and then obtain a classification of matrices. Thanks to this classification we establish a classification of all discrete Hopfield neural networks with n neurons in such a way that two DHNNs belong to the same class if and only if they have the same dynamic property. Lastly, a characterization of the stability of a DHNN with two neurons in synchronous mode is obtained.

1. Introduction

Discrete Hopfield neural networks(DHNNs) were proposed mainly as an associative memory model by Hopfield in [1]. A discrete Hopfield neural network(DHNN) can be viewed as a single layer consisting of n neurons which are connected each other. Each neuron in the networks is in one of two possible states, either 1 or -1. The state of neuron i at time t is denoted by $v_i(t)$. The state of the network at time t is denoted by the vector $v(t) = (v_1(t), v_2(t), \ldots, v_n(t))$.

In recent years, for the purpose of associative memory and combinatorial optimization, several schemes have been established in light of generalized Hopfield neural networks [2-4]. A network can operate in different modes. If the computation is performed at all neurons at the same time, we say that the network is operating in synchronous mode. If the computation is performed only at a single neuron at each time, we say that the network is operating in asynchronous mode. The general form of a DHNN with n neurons in synchronous mode can be described as follows.

$$v_i(t+1) = \mathrm{sgn}^*\left(\sum_{j=1}^{n} w_{ij}v_j(t) - \theta_i\right) (i = 1, 2, \ldots, n) \tag{1.1}$$

where

$$(v_1(t), v_2(t), \ldots, v_n(t)) \in \{-1, 1\}^n \equiv V^n,$$

$$[w_{ij}] \in M_n(\mathbb{R}), (\theta_1, \theta_2, \ldots, \theta_n) \in \mathbb{R}^n,$$

2000 MR Subject Classification: 92B20.
Key words and phrases: Equi-active relation; matrix; stability; Hopfield neural network.
Research partially supported by the NNSF of China (No.19771056).