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Harmonic Foliations on a Complete Riemannian Manifold

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Abstract. Let $\mathcal{F}$ be a Riemannian foliation with finite energy on a manifold (M, g_M) with a complete bundle-like metric g_M. Assume that the Ricci curvature is non-negative and the transversal scalar curvature is non-positive. If $\mathcal{F}$ is harmonic, then $\mathcal{F}$ is totally geodesic.

0 Introduction

A foliation $\mathcal{F}$ on a manifold M is *harmonic*, if the canonical projection $\pi : TM \to Q$ of the tangent bundle to the normal bundle $Q = TM/L$ is a harmonic Q-valued 1-form ([2,3]). For this one needs the connection ∇' defined by (3.10) in Q, and a Riemannian metric g_M in M.

A rich variety of harmonic foliations were discussed in [2]. It is well-known that $\mathcal{F}$ is harmonic if and only if all leaves of $\mathcal{F}$ are minimal submanifolds of M ([2]).

On the other hand, if $\mathcal{F}$ is Riemannian, i.e., if there exists a holonomy invariant metric g_Q on Q, there is a unique metric and torsion-free connection ∇ in Q ([2]).

In 1984, F.W.Kamber and Ph.Tondeur([3]) studied the interplay of the harmonicity property with the curvature of the Riemannian metric g_M and the curvature of the connection ∇, which is metric and trosion-free with respect to the holonomy invariant metric g_Q on Q. Namely, let $\mathcal{F}$ be a Riemannian foliation on a closed oriented manifold M. Let g_M be a Riemannian metric on M with non-negative Ricci curvature and assume the

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