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Weyl Normal Connections of Weyl Submanifolds

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Abstract

We study the Weyl normal connections of Weyl submanifolds. We show that if the Weyl normal connection is flat, then the induced 1-form of a Weyl submanifold is closed and the normal connection is also flat. Next, we investigate a compact Weyl submanifold of an Einstein-Weyl manifold with flat Weyl normal connection.

1. Introduction

Let M^n be a manifold with a conformal structure $[g]$ and a torsion-free affine connection D. A triplet $(M^n,[g],D)$ is called a Weyl manifold if $Dg = \omega \otimes g$ for a 1-form ω. A Weyl manifold $(M^n,[g],D)(n \geqq 3)$ is said to be Einstein-Weyl if the symmetrized Ricci tensor of D is proportional to a representative metric g in $[g]$. A compact Weyl manifold has a unique, up to homothety, metric g in the conformal class such that the 1-form ω is co-closed. We call this metric the Gauduchon metric. If furthermore the manifold is Einstein-Weyl, then the corresponding vector field $\omega^\sharp$ is Killing [11]. Compact Einstein-Weyl manifolds $(M^n,[g],D)(n \geqq 3)$ with closed 1-form ω are classified by Gauduchon in [5]. In [9], Pedersen, Poon and Swann studied Weyl submanifolds of Weyl manifolds. In the previous paper [8], we investigated Weyl space forms and their Weyl submanifolds.

In this paper, we shall study the Weyl normal connections of Weyl submanifolds. Let $(M^n,[g],D)$ be a Weyl submanifold of a Weyl manifold $(\bar{M}^m,[\bar{g}],\bar{D})$ and $\bar{\omega}$ and ω be the corresponding 1-forms of $\bar{g}$ and g respectively. In Section 3, we give the relation between the curvatures of the Weyl normal connection and the normal connection. We show that if the Weyl normal connection is flat, then the induced 1-form ω is closed and the normal connection is also flat. For a hypersurface (M^n,g) isometrically immersed in a Riemannian manifold $(\bar{M}^{n+1},\bar{g})$ the normal connection is flat, but for a Weyl hypersurface $(M^n,[g],D)$ of a Weyl manifold $(\bar{M}^{n+1},[\bar{g}],\bar{D})$ the Weyl normal connection is not necessarily flat. In co-dimension one, we show