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A note on the Grothendieck-Cousin complex on the flag variety in positive characteristic

KANEDA Masaharu

The Grothendieck-Cousin complex of a dominant line bundle on the flag variety with respect to the Schubert filtration is made up of the dual Verma modules in characteristic 0, the dual of a Bernstein-Gelfand-Gelfand complex, as observed by G. Kempf [8], J.L. Brylinski and M. Kashiwara [1], and M. Kashiwara [7], but that does not carry over to positive characteristic. The failure seems not as accessible as the author feels it should be. We intend to remedy the situation by reworking Kashiwara [7], § 3.

The difference stems from the one in SL_2. Thus let K be an algebraically closed field, G the K-group SL_2, B a Borel subgroup of G, T a maximal torus of B, B^+ the Borel subgroup of G opposite to B, α the root of B^+, $W = \langle s_\alpha \rangle$ the Weyl group of G, x_0 the point B of the flag variety $X = G/B$, $\mathcal{L}(\lambda)$ the invertible $\mathcal{O}_X$-module on X induced by a 1-dimensional B-module $\lambda \in \mathrm{Hom}(B, GL_1)$, and $\mathrm{Dist}(G)$ the algebra of distributions of G. In characteristic 0, the $\mathrm{Dist}(G)$-modules $H^1_{B^+ s_\alpha x_0}(X, \mathcal{L}(\lambda))$ and $H^0_{B^+ x_0}(X, \mathcal{L}(s_\alpha \cdot \lambda))$ are isomorphic iff $\langle \lambda, \alpha^\vee \rangle \geq -1$, where $\cdot$ is the dot multiplication and $\alpha^\vee$ is the coroot of α. On the other hand, we will find in § 2 that in positive characteristic they are isomorphic iff $\langle \lambda, \alpha^\vee \rangle = -1$. General results are summarized in (3.4).