Quasi-linear equations of evolution in a Banach space

By

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1. Introductinn and assumptions

In this paper we consider the Cauchy problem for quasi-linear evolution equations of the from

(1) $\quad du/dt+A(t, u)u=F(t, u),\ 0\leq t\leq T,$

(2) $\quad u(0)=\varphi$

in an arbitrary Banach space X. Here $A(t, u)$ is, for every $t\in[0, T]$ and $u\in X$, not necessarily a bounded linear operator acting in X and the given function $F(t, u)$ takes values in X.

Under some assumptions on $A(t, u)$ and $F(t, u)$ specified below, making use of the semi-group theory, we try to solve (1)-(2) by successive approximation method in the following manner:

$$\begin{cases} du_0/dt+A(t, \varphi)u_0=F(t, \varphi),\ 0\leq t\leq T,\quad u_0(0)=\varphi; \\ du_{m+1}/dt+A(t, u_m)u_{m+1}=F(t, u_m),\ 0\leq t\leq T,\ u_{m+1}(0)=\varphi,\ m=0, 1, \ldots. \end{cases}$$

Assuming nothing on X itself and constructing step by step the fundamental solutions of the above equations by method of H. Tanabe [2], we can obtain a sequence of uniformly bounded functions $u_m(t)$, $m=0, 1, \ldots$ whose strong limit exists and gives a desired solution* in some interval $[0, T_0]$ $(0<T_0(\varphi)\leq T)$ for an arbitrary φ in the domain $D=D(A(t, u))$ of $A(t, u)$. Next, we shall prove uniqueness and continuous dependence on the initial value of the solution $u(t)$.

Throughout this note, we shall make the following assumptions:

$(\mathrm{A})_\varphi$ For each $t\in[0, T]$ and $u\in X$, $A(t, u)$ is a closed linear operator, whose domain D is dense in X and independent of t and u, and fulfills

$$(\mathrm{I})_\varphi \quad \|\{A(t, u)-A(t, v)\}A_\varphi(t)^{-1}\|\leq f_\varphi(\|u\|+\|v\|)\|u-v\|,$$

$$(\mathrm{II})_\varphi \quad \|\{A(t, u)-A(s, u)\}A_\varphi(s)^{-1}\|\leq g_\varphi(\|u\|)|t-s|$$

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* We call u a solution of (1)-(2) in $[0, T]$ if u belongs to D, is strongly continuously differentiable on $[0, T]$ and satisfies (1)-(2).