Certain Invariant Subspace Structure of Analytic Crossed Products II

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1. Introduction

Let M be a finite von Neumann algebra on a separable Hilbert space H. Let α be a $*$-automorphism of M. Suppose that there is an α-invariant faithful normal semi-finite trace ϕ of M. Let $\mathfrak{L}_+$ be an analytic crossed product on $\boldsymbol{L}^2$ determined by M and α (see the definition to § 2). We have an interest in the invariant subspace structure of $\boldsymbol{L}^2$ with respect to $\mathfrak{L}_+$. In [**3, 5**], McAsey introduced the notion of canonical models for invariant subspaces of $\boldsymbol{L}^2$. That is, a family of left-pure, left-full, left-invairant subspaces $\{\mathfrak{M}_i\}_{i\in I}$ constitutes a complete set of canonical models for all invariant subspaces of $\boldsymbol{L}^2$ in case (a) for no two distinct indices i and j, $P_{\mathfrak{M}_i}$ is unitary equivalent to $P_{\mathfrak{M}_j}$ by a unitary operator in $\mathfrak{R}(=\mathfrak{L}')$; and (b) for every left-pure, left-invariant subspace of $\boldsymbol{L}^2$, there is an i in I and a partial isometry V in $\mathfrak{R}$ such that $VP_{\mathfrak{M}_i}V^*=P_{\mathfrak{M}}$, so that $\mathfrak{M}=V\mathfrak{M}_i$. McAsey found the canonical model in case that $M=\ell^\infty(X)$, where X is a finite set with elements $t_0, t_1, \dots, t_{K-1}$, and the automorphism of M induced by a permutation of X. Further, in [**9, 18, 19**], we studied the canonical models of invariant subspaces in case that ϕ is a finite trace. On the other hand, in [**2**], we studied the canonical models when $M=L^\infty(X, \mu)$, $\mu(X)=\infty$ and α is an ergodic automorphism of M. That is, we constructed a left-pure, left-full, left-invariant subspace $\mathfrak{M}_\infty$ of $\boldsymbol{L}^2$ with the multiplicity function m of $\mathfrak{M}_\infty$ which is $m(t)=\infty$ for almost all t in X. Thus, for every left-pure, left-invariant subspace $\mathfrak{M}$ of $\boldsymbol{L}^2$, there exists a partial isometry V in $\mathfrak{R}(=\mathfrak{L}')$ such that $\mathfrak{M}=V\mathfrak{M}_\infty$. That is, the canonical model in this case is the singletone $\{\mathfrak{M}_\infty\}$.

Our aim in this note is to extend the results in [**2**]. That is, suppose that $\phi(1)=\infty$ and that α is ergodic on the center Z of M. Then we will construct a left-pure, left-full, left-invariant subspace $\mathfrak{M}_\infty$ of $\boldsymbol{L}^2$ with the multiplicity function m of $\mathfrak{M}_\infty$ which is $m(t)=\infty$ for almost everywhere t in X. Therefore, we prove that for every left-pure, left-invariant subspace $\mathfrak{M}$ of $\boldsymbol{L}^2$, there exists a partial isometry V in $\mathfrak{R}(=\mathfrak{L}')$ such that $\mathfrak{M}=V\mathfrak{M}_\infty$.

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