On a Galton-Watson process with state-dependent immigration

By

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1. Introduction

Consider a Galton-Watson process in which immigration is allowed in a generation if the number of the previous generation is smaller than or equal to i_0, where i_0 is a fixed positive integer.

Denote the size of the n-th generation by X_n, and we set up this process as follows;

1) $A(x)=\sum_{j=0}^{\infty} a_j x^j$, $(|x|\leqq 1)$, is the probability generating function of the offspring distribution;

2) $B_k(x)=\sum_{j=0}^{\infty} b_{kj}x^j$, $(|x|\leqq 1,\ 0\leqq k\leqq i_0)$, is the probablity generating function of the immigrants distribution, where b_{kj} is the probability that j immigrants enter in a generation when the number of the previous generation is equal to k;

3) transition probability p_{ij} is given by

$$
\begin{aligned}
p_{ij} \equiv P\{X_{n+1}=j\,|\,X_n=i\} &= a_j^{(i^*)}, \qquad i\geqq i_0+1, \\
&= \sum_{k=0}^{j} b_{ik}\cdot a_{j-k}^{(i^*)}, \qquad 0\leqq i\leqq i_0,
\end{aligned} \tag{1}
$$

where $a_j^{(i^*)}$ is the j-th term in the i-th fold convolution of the sequence $\{a_j\}$.
The state-dependent immigration have been studied by Pakes (1971) [9], Foster (1971) [4] and Nakagawa-Sato (1974) [8]. The setting up of Pakes or Foster is the case that $B_0(x)=B(x)$ and $B_k(x)=1 (k=1, 2, \cdots, i_0)$, and Nakagawa-Sato's is the case that $B_k(x)=B(x)$ for all k, in our process.

From now on it will be assumed that

1. $0<a_0,\ a_0+a_1,\ b_{k0}<1, \qquad (k=0, 1, \cdots\cdots, i_0)$;
2. $\alpha=A'(1-)<\infty$ and $\beta_k=B_k'(1-)\leqq\infty, \qquad (k=0, 1, \cdots\cdots, i_0)$.