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Another Proof of decomposability of Nambu-Poisson tensors

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Abstract. Although Nambu-Poisson bracket is a natural generalization of Poisson bracket, a very distinguished property of Nambu-Poisson bracket comparing Poisson bracket is decomposability of its tensor. This is first conjectured in [1] and is given affirmative answers by [2] and [4] independently. In this paper, we shall show another proof to decomposability of Nambu-Poisson tensor, which is more elementary and more direct to the property of decomposability comparing that of [2] or [4].

1 Introduction

In contrast to Poisson bracket being a binary operation, Nambu-Poisson is a multi-fold operation provided with the same properties of Poisson bracket and the fundamental identity which is a natural generalization of Jacobi identity. We recall the precise definition of Nambu-Poisson bracket. Let M be a n-dimensional C^∞-manifold. An order p Nambu-Poisson bracket on M is a p-fold skew-symmetric $\mathbf{R}$-multilinear operation

$$\{\dots\} : C^\infty(M)^p := \underbrace{C^\infty(M) \times \cdots \times C^\infty(M)}_{p\text{-times}} \longrightarrow C^\infty(M)$$

provided with Leibniz rule for each argument, and the fundamental identity (or generalized Jacobi identity):

$$\{\mathcal{F}, \{\mathcal{G}\}\} = \sum_{\ell=1}^{p} \{g_1, \dots, \{\mathcal{F}, g_\ell\}, \dots, g_p\}$$

where $\mathcal{F} = (f_1, \dots, f_{p-1}) \in C^\infty(M)^{p-1}$, $\mathcal{G} = (g_1, \dots, g_p) \in C^\infty(M)^p$.
If order $p = 2$, then the fundamental identity is just Jacobi identity and order 2 Nambu-Poisson brackets are Poisson brackets. Like as Poisson brackets, every order p Nambu-Poisson bracket

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