Nihonkai Math. J.
Vol.9 (1998), 155–169

Von Neumann-Jordan constant and uniformly non-square Banach spaces

Yasuji TAKAHASHI* and Mikio KATO†

Abstract. A sequence of characterizations of uniform non-squareness is given, some of which are similar to the well-known homogeneous characterization of uniformly convex spaces. As corollaries: (i) Banach spaces with von Neumann-Jordan constant less than 2 are characterized as those uniformly non-square; (ii) it is presented that uniform non-squareness is inherited by dual spaces.

1. Introduction and preliminaries

According to Clarkson [5] the *von Neumann-Jordan (NJ-) constant* of a Banach space X, we denote it by $C_{NJ}(X)$, is the smallest constant C for which

$$\frac{1}{C} \leqq \frac{\| x + y \|^2 + \| x - y \|^2}{2(\| x \|^2 + \| y \|^2)} \leqq C \qquad (1)$$

holds for all x, y $\in$ X with $\| x \|^2 + \| y \|^2 \neq 0$. (Note that the first and second inequalities of (1) are equivalent; put x + y = u, x − y = v.) Classical results state that: (i) $1 \leqq C_{NJ}(X) \leqq 2$ for any Banach space

, †) Supported in part by Grants-in-Aid for Scientific Research from the Ministry of Education, Science and Culture (09640214 resp. 09640203†).