

ORLICZ NORM ESTIMATES FOR POISSON MAXIMAL OPERATORS

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ABSTRACT. A condition for Poisson maximal operator to be of weak type L_ϕ are characterized in terms of the Orlicz norm. This operator unifies various maximal operators cited in the literatures.

I. Introduction

For a given function f on \mathbb{R}^n , set

$$\mathcal{M}f(x, t) = \sup_Q \frac{1}{|Q|} \int_Q |f| dx, \quad (x \in \mathbb{R}^n, t \geq 0),$$

where the supremum is taken over the cubes Q in \mathbb{R}^n centered at x with sides parallel to the x -axis and has side length at least t . It is well known that this operator plays an important role in studying the Poisson integral on the upper half-space.

For a given positive measure ν on $\mathbb{R}^n \times [0, \infty)$, under what condition on ν can we assert that \mathcal{M} is bounded from $L^p(\mathbb{R}^n)$ into $L^p(\mathbb{R}^n \times [0, \infty), \nu)$? Carleson[C] showed this is equivalent to the Carleson condition and later Fefferman-Stein[FS] found a sufficient condition, and later Ruiz[R], and Ruiz-Torrea[RT] unified all these results. Further, Gallardo[G] and Chen[Ch] obtained a characterization in terms of the Orlicz norm.

On the other hand, Sueiro[Su] studied a maximal operator \mathcal{M}_Ω to study Poisson-Szegö integral. This operator generalizes the standard Hardy-Littlewood maximal operator.

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