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SPECTRAL MAPPING THEOREMS AND WEYL SPECTRA FOR HYPONORMAL OPERATORS

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Abstract

In this paper, we will give an elementary proof of Lemma VI.4.2 of [6] and show that the spectral mapping theorem holds for Weyl spectra of this mapping.

1. Introduction.

Let $\mathcal{H}$ be a complex Hilbert space and $B(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. An operator $T \in B(\mathcal{H})$ is said to be hyponormal if $T^*T \geq TT^*$. For an operator T, we denote the spectrum and the approximate point spectrum by $\sigma(T)$ and $\sigma_a(T)$, respectively. A point $z \in \mathbf{C}$ is in the joint approximate point spectrum $\sigma_{ja}(T)$ if there exists a sequence of unit vectors $\{x_n\}$ in $\mathcal{H}$ such that $(T-z)x_n \to 0$ and $(T-z)^*x_n \to 0$. In [6] D. Xia proved the following result:

Theorem A (Lemma VI.4.2 of [6]). *Let $T = H + iK$ be hyponormal and f, g be bounded real-valued, continuous functions and $f(x) \neq 0$. Take a mapping in the complex plane*

$$\tau(x + iy) = x + i(f(x)^2 y + g(x))$$

and denote $\tau(T) = H + i(f(H)Kf(H) + g(H))$. Then

$$\sigma(\tau(T)) = \tau(\sigma(T)).$$

This proof needs the singular integral model of a hyponormal operator. In this paper we will give an elementary proof of the following theorem without the singular integral model.

Theorem 1. *Let $T = H + iK$ be hyponormal and f, g be bounded real-valued, continuous functions and $f(x) \neq 0$ at $x \in \sigma(H)$. Take a mapping in the complex plane*

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