ON PSEUDO-UMBILICAL SURFACES WITH NONZERO PARALLEL MEAN CURVATURE VECTOR IN $\mathbb{C}P^3(\tilde{c})$

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Abstract. Any pseudo-umbilical surface with nonzero parallel mean curvature vector in $\mathbb{C}P^3(\tilde{c})$ is a totally real isotropic surface in $\mathbb{C}P^3(\tilde{c})$.

1. Introduction

Let $\mathbb{C}P^m(\tilde{c})$ be a complex m-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature \tilde{c} .

Chen and Ogiue [1] classified totally umbilical submanifolds in $\mathbb{C}P^m(\tilde{c})$. However, it is well known that the class of pseudo-umbilical submanifolds in $\mathbb{C}P^m(\tilde{c})$ is too wide to classify. Thus, it is reasonable to study pseudo-umbilical submanifolds in $\mathbb{C}P^m(\tilde{c})$ under some additional condition.

Recently, the author proved that any pseudo-umbilical submanifold M^n with nonzero parallel mean curvature vector in $\mathbb{C}P^m(\tilde{c})$ is a totally real submanifold and satisfies 2m > n ([3]). Thus, we see that $\mathbb{C}P^2(\tilde{c})$ admits no pseudo-umbilical surfaces with nonzero parallel mean curvature vector.

In the previous paper [4], the author showed that any complete pseudo-umbilical isotropic surface of $P(\mathbb{R})$ -type (see Preliminaries) with nonzero parallel mean curvature vector in $\mathbb{C}P^4(\tilde{c})$ is an extrinsic hypersphere in a 3-dimensional real projective space $\mathbb{R}P^3(\tilde{c}/4)$ of $\mathbb{C}P^3(\tilde{c})$.

The aim of this paper is to prove the following result.