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A NEW CHARACTERIZATION OF HOMOGENEOUS REAL HYPERSURFACES IN COMPLEX SPACE FORMS

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ABSTRACT. The purpose of this paper is to give a new characterizations of homogeneous real hypersurfaces M in complex space forms $M_n(c)$ when the covariant derivative and the Lie derivative of the Ricci tensor of M are equal to each other along the direction of the structure vector ξ.

1. Introduction

A complex n-dimensional Kähler manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. The complete and simply connected complex space form is isometric to a complex projective space P_nC, a complex Euclidean space C^n, or a complex hyperbolic space H_nC according as $c > 0$, $c = 0$ or $c < 0$ respectively. The induced almost contact metric structure of a real hypersurface M of $M_n(c)$ is denoted by (ϕ, ξ, η, g).

Now, there exist many studies about real hypersurfaces of $M_n(c)$, $c \neq 0$. One of the first researches is the classification of homogeneous real hypersurfaces of a complex projective space P_nC by Takagi [14], who showed that these hypersurfaces of P_nC could be divided into six types which are said to be of type A_1, A_2, B, C, D and E. This result is generalized by many authors (See [3], [5], [8], [9], [11] and [13]).

On the other hand, real hypersurfaces of H_nC have been also investigated by many authors (See [1], [6], [10] and [12]) from different points of view. In particular, Berndt [1] proved the following.

Theorem A. *Let M be a real hypersurface of H_nC, $n \geqq 3$. Then M has constant principal curvatures and ξ is principal if and only if M is locally congruent to one of the followings :*

(A_0) *a horosphere in H_nC, that is, a Montiel tube,*
(A_1) *a tube over a totally geodesic hyperplane H_kC ($k = 0$ or $n-1$),*
(A_2) *a tube over a totally geodesic H_kC ($1 \leqq k \leqq n-2$),*

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