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ON WEYL SPECTRUM AND A CLASS OF OPERATORS

YOUNGOH YANG

ABSTRACT. In this paper we show that the set $\mathcal{W}$ of all operators satisfying the equality of the Weyl and essential spectra is norm closed in $B(H)$, invariant under compact perturbation, and closed under approximate similarity. But $\mathcal{W}$ is not closed under addition. Also we show that the Weyl spectrum of an operator in $\mathcal{W}$ satisfies the spectral mapping theorem for analytic functions and give properties of an operator in $\mathcal{W}$.

0. Introduction. Let H be an infinite dimensional Hilbert space and we write $B(H)$ for the set of all bounded linear operators on H and $\mathcal{K}$ for the set of all compact operators on H. If $T \in B(H)$, we write $\sigma(T)$ for the spectrum of T and $\pi_{00}(T)$ for the isolated points of $\sigma(T)$ which are eigenvalues of finite multiplicity. An operator $T \in B(H)$ is said to be *Fredholm* if its range ran T is closed and both the null space ker T and ker T^* are finite dimensional. The *index* of a Fredholm operator T, denoted by $i(T)$, is defined by

$$i(T) = \dim\ker T - \dim\ker T^*.$$

It was well-known ([4]) that $i : \mathcal{F} \to \mathbb{Z}$ is a continuous function where the set $\mathcal{F}$ of Fredholm operators has the norm topology and $\mathbb{Z}$ has the discrete topology. The *essential spectrum* of T, denoted by $\sigma_e(T)$, is defined by

$$\sigma_e(T) = \{\lambda \in \mathbb{C} : \ T - \lambda I \text{ is not Fredholm}\}.$$

A Fredholm operator of index zero is called *Weyl.* The *Weyl spectrum* of T, denoted by $\omega(T)$, is defined by

$$\omega(T) = \{\lambda \in \mathbb{C} : \ T - \lambda I \text{ is not Weyl}\}.$$

It was shown ([1]) that for any operator T, $\sigma_e(T) \subset \omega(T) \subset \sigma(T)$ and equalities do not hold in general. Also

$$\omega(T) = \cap_{K \in \mathcal{K}} \sigma(T + K)$$

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