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A strong convergence theorem for an iteration of nonexpansive mappings

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1 Introduction

Let C be a nonempty closed convex subset of a real Banach space E. Then, a mapping T of C into itself is said to be nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\| \qquad \textit{for all } x, y \in C.$$

We deal with the following iterative process, first considered by Halpern[2]:

$$A_0^\alpha x = x, \quad A_{n+1}^\alpha x = \alpha_{n+1} x + (1 - \alpha_{n+1}) T A_n^\alpha x \ (n = 0, 1, 2, \cdots), \qquad (1)$$

where $\alpha_n \in [0, 1]$. Recently, Wittmann[5] proved a strong convergence theorem of iterates $\{A_n^\alpha x\}$ defined by (1) in the case when E is a Hilbert space and $\{\alpha_n\}$ satisfies $0 \leq \alpha_n \leq 1$, $\lim_{n\to\infty} \alpha_n = 0$, $\sum_{n=1}^{\infty} \alpha_n = +\infty$ and $\sum_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < +\infty$; see[3].

In this paper, we extend Wittmann's result to a uniformly convex and uniformly smooth Banach space with a weakly sequentially continuous duality mapping.

2 Preliminaries

Let E be a real Banach space, and let $S_1[0] = \{x \in E : \|x\| = 1\}$ be its unit sphere. The norm of E is said to be Gâteaux diffrentiable (and E is said to be smooth), if

$$\lim_{t\to 0} \frac{\|x + ty\| - \|x\|}{t} \qquad (2)$$