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# SEMINORMAL OPERATORS AND WEYL SPECTRA

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ABSTRACT. In this paper we show that the Weyl spectrum of a seminormal operator $T$ satisfies the spectral mapping theorem for any analytic function $f$ on a neighborhood of $\sigma(T)$ and Weyl's theorem holds for $f(T)$. Finally we give conditions for an operator to be of the form unitary + compact and answer an old question of Oberai.

**0. Introduction.** Throughout this paper let $H$ denote an infinite dimensional Hilbert space and $B(H)$ the set of all bounded linear operators on $H$. If $T \in B(H)$, we write $\sigma(T)$ for the spectrum of $T$, $\pi_0(T)$ for the set of eigenvalues of $T$, $\pi_{0f}(T)$ for the set of eigenvalues of finite multiplicity, and $\pi_{00}(T)$ for the isolated points of $\sigma(T)$ that are eigenvalues of finite multiplicity. If $E$ is a subset of $\mathbb{C}$, we write iso $E$ for the set of isolated points of $E$. An operator $T \in B(H)$ is said to be *Fredholm* if its range ran $T$ is closed and both the null spaces ker $T$ and ker $T^*$ are finite dimensional. The *index* of a Fredholm operator $T$, denoted by $i(T)$, is defined by

$$i(T) = \dim\ker T - \dim\ker T^*.$$

The *essential spectrum* of $T$, denoted by $\sigma_e(T)$, is defined by

$$\sigma_e(T) = \{\lambda \in \mathbb{C} : \ T - \lambda I \text{ is not Fredholm}\}.$$

A Fredholm operator of index zero is called a *Weyl* operator. The *Weyl spectrum* of $T$, denoted by $\omega(T)$, is defined by

$$\omega(T) = \{\lambda \in \mathbb{C} : \ T - \lambda I \text{ is not Weyl}\}.$$

It was shown ([2]) that for any operator $T$, $\sigma_e(T) \subset \omega(T) \subset \sigma(T)$, and $\omega(T)$ is a nonempty compact subset of $\mathbb{C}$.

Recall ([9], [12]) that an operator $T \in B(H)$ is said to be *seminormal* if either $T$ or $T^*$ is hyponormal. Every hyponormal operator is seminormal,

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