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Conjugacy classes of zero entropy automorphisms on free group factors

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1. Introduction. The entropy $H(\theta)$ of a $*$-automorphism θ on a von Neumann algebra M is defined by Connes - Størmer [4] as an extended version of classical one. The notion of entropy is conjugacy invariant, that is, $H(\theta) = H(\alpha^{-1}\theta\alpha)$ for an automorphism α of M.

Besson[2] gives an example of an uncountable family of automorphisms on the hyperfinite II_1 factor R which have zero entropy but are not pairwize conjugate. An interesting example of II_1-factor which is not hyperfinite is the group von Neumann algebra $L(F_n)$ of the free group F_n on n generators ($n \geq 2$).

The purpose of this paper is to give an alternative version of Besson's result to free group factors. That is, we show :

Theorem. *There exists an uncountable family of automorphisms on $L(F_n)$ which have entropy zero but are pairwize non conjugate.*

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2. Automorphisms of free group factors. Let G be a countable infinite group and $l^2(G)$ the Hilbert space of all square summable functions on G. For each g in G, let $u(g)$ be the unitary representation of G to $l^2(G)$ defined by

$$(u(g)\xi)(h) = \xi(g^{-1}h) \qquad (\xi \in l^2(G),\, h \in G).$$

The von Neumann algebra on $l^2(G)$ generated by $\{u(g); g \in G\}$ is called the left von Neumann algebra of G and denoted by $L(G)$. It is well known that L(G) is factor if and only if G is an ICC group, that is, every conjugacy class $C_g = \{hgh^{-1}; h \in G\}$ is infinite, except the trivial $\{1\}$. Let $\{\delta(g)\}_{g\in G}$ be an othonomal basis in $l^2(G)$ given by

$$(\delta(g))(h) = \begin{cases} 1 & h = g \\ 0 & \text{otherwise} \end{cases} \qquad (g \in G).$$

The functional τ on $L(G)$ defined by

$$\tau(x) = (x\delta(e)|\delta(e)) \qquad (x \in R(G),\, e \text{ is the unit of } G),$$