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# Pareto Optimum in a Cooperative Dynkin's Stopping Problem [1]

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**Abstract.** We consider two-person cooperative stopping gameof Dynkin's type and find $\varepsilon$–Pareto optimal pairs of stopping times by three methods. The first is the so–called scalarization and the corresponding optimal value process is characterized by a recursive relation. In the second we find an $\varepsilon$–Pareto optimal pair nearest to a goal, which two players desire but may not be able to achieve. We select thirdly a Pareto optimal pair which dominates a conservative value for each player. The set of such Pareto optimal pairs is called core. We finally apply them to a Markov model and give simple examples.

**Key words.** stopping game, cooperative game, Pareto optimal, martingale, core

## 1. Introduction.

Let $(\Omega, \mathcal{F}, P)$ be a probability space and $(\mathcal{F}_n)_{n\in N}$ an increasing family of sub-$\sigma$-fields of $\mathcal{F}$, where $N = \{0,1,2,\ldots\}$ is a time space. Let $\mathcal{W}$ denote the family of all sequences of random variables $X = (X_n)_{n\in N}$ defined on $(\Omega, \mathcal{F}, P)$ and adapted to $(\mathcal{F}_n)$ such that random variable $\sup_n X_n^+$ is integrable and sequence $(X_n^-)$ is uniformly integrable, where $x^+ = \max(0,x)$ and $x^- = (-x)^+$. For each $n \in N$, we also denote by $\Lambda_n$ the class of pairs of $(\mathcal{F}_n)$–stopping times $(\tau, \sigma)$ such that $n \leqq \tau \wedge \sigma < \infty$ a. s. , where $\tau \wedge \sigma = \min(\tau, \sigma)$.

For six random sequences $X^i, Y^i$ and $W^i$ $(i = 1,2)$ in $\mathcal{W}$, we consider the following cooperative stopping game with a finite constraint. There are two players and the first and the second players choose stopping times $\tau_1$ and $\tau_2$, respectively, such that $(\tau_1, \tau_2)$ is in $\Lambda_0$. Then the $i$th player $(i = 1,2)$ gets the reward

$$J_i(\tau_1,\tau_2) = X^i_{\tau_i} I_{(\tau_i<\tau_j)} + Y^i_{\tau_j} I_{(\tau_j<\tau_i)} + W^i_{\tau_i} I_{(\tau_i=\tau_j)}, \quad j = 1,2,\ j \neq i,$$

where $I_A$ is the indicator function of a set $A$ in $\mathcal{F}$. The aim of the $i$th player is to maximize the expected gain $E[J_i(\tau_1,\tau_2)]$ with respect to $\tau_i$, cooperating with another player, if necessary. However, the stopping time chosen by one of them generally depends upon one decided by another, even if they cooperate. Thus we shall use the concept of Pareto optimality as in the usual cooperative game or the multiobjective problem.

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