

On $*$ -Representations of Partial $*$ -Algebras

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Abstract

The first purpose of this paper is to study $*$ -subrepresentations of a $*$ -representation of a partial $*$ -algebra. The second purpose is to characterize invariant positive sesquilinear forms of type I, II, III.

1. Introduction.

In this paper we shall investigate the fundamental properties of $*$ -representations of partial $*$ -algebras. The study of $*$ -representations of partial $*$ -algebras and partial O^* -algebras were began by Antoine and Karwowski [1], and have been continued by Antoine, Inoue and Trapani [2], from the situation of pure mathematical and the physical applications. But, the studies of $*$ -subrepresentations and invariant positive sesquilinear forms on partial $*$ -algebras seem to be insufficient, and so we shall study these points in this paper.

In partial $*$ -algebras, the multiplication is defined only partially and it dose not have the associative low. And so, to extend arguments that are considered in the case of $*$ -algebras, we need to reconsider some conditions. For example, the quasi-weak commutant $C_{\text{qw}}(\pi)$ is considered instead of the usual weak commutant $C_w(\pi)$ of π .

Let π be a $*$ -representation of a partial $*$ -algebra \mathcal{A} . For each projection E in $C_{\text{qw}}(\pi)$, we can define the $*$ -representation π_E of \mathcal{A} by

$$\mathcal{D}(\pi_E) := E\mathcal{D}(\pi), \quad \pi_E(x) := \pi(x)E \quad (x \in \mathcal{A}).$$