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# MEAN INTEGRATED SQUARED ERROR AND DEFICIENCY OF NONPARAMETRIC RECURSIVE KERNEL ESTIMATORS OF SMOOTH DISTRIBUTION FUNCTIONS

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## Abstract

In this paper we derive asymptotic expressions of the mean integrated squared error (MISE) as a global measure for a nonparametric recursive kernel estimator of a distribution function. We also investigate whether the recursive kernel estimator has asymptotically better performance than the empirical distribution function. It is proved that the relative deficiency of the empirical distribution function with respect to an appropriately chosen recursive kernel estimator quickly tends to infinity as the sample size increases.

## 1. Introduction

Let $X_1, X_2, \ldots$ be independent and identically distributed random variables having a common cumulative distribution function (c.d.f) $F$. Throughout this paper, the c.d.f $F$ is assumed to be absolutely continuous with respect to Lebesgue measure, and let a probability density function (p.d.f) of $F$ be denoted by $f$.

In this paper, we consider the problem of estimating the c.d.f $F$. Traditionally, as an estimator of $F$, the empirical distribution function

$$F_n(x) = \frac{1}{n}\sum_{j=1}^{n} I(X_j \le x) \tag{1.1}$$

has been chosen, where $I(\cdot)$ denotes the indicator function. It is well known that the estimator is strongly uniformly consistent. But this estimator does not take into account the smoothness of $F$, that is, the existence of the density $f$. On the other hand, taking into account this point, alternative estimators of the c.d.f $F$ have been considered by (to mention a few) Nadaraya [7], Yamato [17], Winter [15, 16], Hill [3], Puri and Ralescu [8], Yukitch [18] and Sadra [11]. Especially, for kernel-type estimators, Singh et al. [13] investigated their asymptotic properties including strong uniform consistency and asymptotic normality, and also derived rates of convergence.

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