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# Regularity of the solutions of some hypoelliptic operators

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**Abstract.** Let $P(D)$ be an hypoelliptic operator with constant coefficients, having a fundamental solution that is locally integrable in $\mathbb{R}^n$. Let $u$ be a distribution defined on an open set $\Omega$ in $\mathbb{R}^n$ such that $Pu = f$. It's proved that if $f \in L^1_{loc}(\Omega)$ then $u \in L^1_{loc}(\Omega)$ and if $f$ is in $C^m(\Omega)$ so is $u$.

## 1. Introduction.

Let $\Omega$ be a domain in $\mathbb{R}^n$, $n \geq 1$. Let $A = \sum_{|\alpha|\leq m} a_\alpha(x)D^\alpha$ be a differential operator of order $m$ with $a_\alpha \in C^m(\Omega)$. Let $A^*$ denote the adjoint operator of $A$. Let $1 < p \leq \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. If $f \in L^p(\Omega)$, it is proved in [1], there exists a weak solution of $Au = f, u \in L^p(\Omega)$ and $\|u\|_p \leq c$ if and only if $| < f, \phi > | \leq c\|A^*\phi\|_q$ for all $\phi \in C_0^\infty(\Omega)$. In this note we discuss the possibility of finding an $L^1_{loc}(\Omega)$ solution $u$ for the equation $Au = f$, if it is known that $f \in L^1_{loc}(\Omega)$.

Now it is known that if $P = \sum_{|\alpha|\leq m} a_\alpha D^\alpha$ is a hypoelliptic differential operator of order $m$ with constant coefficients and $\Omega$ is a convex open set of $\mathbb{R}^n$, then for any $T \in D'(\Omega)$, there exists a distribution $u \in D'(\Omega)$ such that $Pu = T$ (see [2]). If we suppose moreover that $P$ is elliptic, then the above result is true even if $\Omega$ is

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